RESOLUTIONS OF SINGULARITIES.

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The general reference has been [2]. See also an article [1](arXiv:math/0211423)

Definition 7.1. Let W be a scheme. The *order* of an ideal I at $a \in W$ is defined as follows.

$$\operatorname{ord}_a I = \max\{k | \mathfrak{m}_a^k \supset I\}$$

The *top locus* $\operatorname{top}(t)$ of an upper semicontinuous function t on V is a set of points of V where t attains its maximum. We put:

$$\operatorname{top}(I) = \operatorname{top}(\operatorname{ord}(I))$$

We consider an variety X contained in a regular variety W. We let $J = J_X$ the defining ideal of X. We decompose:

$$J = M \cdot I$$

M is the "resolved part", whereas I is the "unresolved part", of the ideal J.
Objective: By blowing up several times, reduce the order o of the ideal I.

We need to find the center Z of the blowing up. It is given as a top locus $\operatorname{top}(i_a)$ of a certain function $i_\bullet = i_a$.

We now introduce a result which is specific to the characteristic zero case.

Lemma 7.2. *For any point $a \in W$, there exists a local hyper surface $V \subset W$ ("a hyper surface of maximal contact") such that "blow ups in the center in V conatins all the equiconstant points."*

By using the lemma above, we develop an inductive argument on the dimension. Namely, by using the theory of "coefficient ideals", we define an ideal J_- in V.

There are two problems:

(1) Z_- may not be transversal to the exceptional locus F. Additional "small" blow ups (along with, say, Q) are needed.
(2) "Blow ups" and $\bullet_-$ may not commute. Therefore we need to decompose $J = M \cdot I$ and see how M and I change.

The function i_a is then be defined (inductively) by:

$$i_a(J) = (\operatorname{ord}_a(I), \operatorname{ord}_a(Q), m_a, i_a(J_-)).$$

References

[1] Herwig Encinas, S.; Hauser, *Strong resolution of singularities in characteristic zero*, Comment. Math. Helv. **77** (2002), no. 4, 821–.

[2] Herwig Hauser, *The Hironaka theorem on resolution of singularities (or: A proof we always wanted to understand).*, Bull. Am. Math. Soc., New Ser. **40** (2003), no. 3, 323–403 (English).