RESOLUTIONS OF SINGULARITIES.

YOSHIFUMI TSUCHIMOTO

02. Affine varieties, projective spaces and projective varieties.

Definition 2.1. Let $\mathbb{k}$ be a field. For any $f_1, \dots f_m \in \mathbb{k}[X_1, \dots, X_n]$, we put

$$V(f_1, \dots, f_m)(\mathbb{k}) = \{x \in \mathbb{k}^n; f_1(x) = 0, \dots, f_m(x) = 0\}$$

and call it (the set of $\mathbb{k}$-valued point of) the **affine variety** defined by $\{f_1, f_2, \dots, f_m\}$.

Definition 2.2. Let R be a ring. A polynomial $f(X_0, X_1, \dots, X_n) \in R[X_0, X_1, \dots, X_n]$ is said to be **homogenius** of degree d if an equality

$$f(\lambda X_0, \lambda X_1, \dots, \lambda X_n) = \lambda^d f(X_0, X_1, \dots, X_n)$$

holds as a polynomial in $n + 2$ variables $X_0, X_1, X_2, \dots, X_n, \lambda$.

Definition 2.3. Let $\mathbb{k}$ be a field.

(1) We put

$$\mathbb{P}^n(\mathbb{k}) = (\mathbb{k}^{n+1} \setminus \{0\})/\mathbb{k}^\times$$

and call it (the set of $\mathbb{k}$-valued points of) the **projective space**. The class of an element $(x_0, x_1, \dots, x_n)$ in $\mathbb{P}^n(\mathbb{k})$ is denoted by $[x_0 : x_1 : \cdots : x_n]$.

(2) Let $f_1, f_2, \dots, f_l \in \mathbb{k}[X_0, \dots, X_n]$ be homogenious polynomials. Then we put

$$V_h(f_1, \dots, f_l) = \{[x_0 : x_1 : x_2 : \dots x_n]; f_j(x_0, x_1, x_2, \dots, x_n) = 0 \qquad (j = 1, 2, 3, \dots, l)\}.$$

and call it (the set of $\mathbb{k}$-valued point of) the **projective variety** defined by $\{f_1, f_2, \dots, f_l\}$.

(Note that the condition $f_j(x) = 0$ does not depend on the choice of the representative $x \in \mathbb{k}^{n+1}$ of $[x] \in \mathbb{P}^n(\mathbb{k})$.)