RESOLUTIONS OF SINGULARITIES.

YOSHIFUMI TSUCHIMOTO

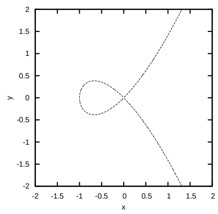
01. Nagata Hironaka.

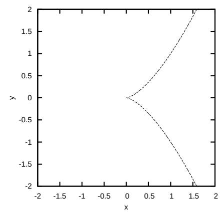
We refer to [1] as a general reference.

Theorem 1.1 (Nagata). *Every scheme X of finite type over a Noetherian integral scheme S is a open subscheme of a proper scheme X' over S. of a complete*

Theorem 1.2 (Hironaka). *Every singular variety over a field $\Bbbk$ of characteristic 0 has a "resolution of singularities".*

Examples of singular curves: $y^2 = x^2 + x^3$, $y^2 = x^3$

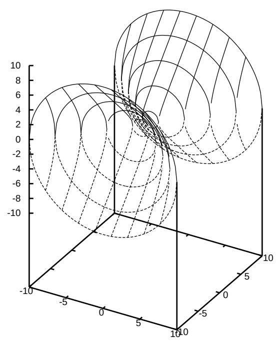




Blow up: Add an extra variable $u = y/x$.
Sometimes we need to blow up several times to obtain regular curve.

Exercise 1.1. Resolve the singularity of $y^3 = x^5$.

An example of a singular surface: $z^2 = y^2 - x^2$



Blow up: Add extra variables $u = y/x, v = z/x$.

References

[1] Herwig Hauser, *The Hironaka theorem on resolution of singularities (or: A proof we always wanted to understand).*, Bull. Am. Math. Soc., New Ser. **40** (2003), no. 3, 323–403 (English).