ZETA FUNCTIONS. NO.10

YOSHIFUMI TSUCHIMOTO

Problems

According to Proposition 5.2, a congruent zeta function of an elliptic curve E is given as follows.

$$\frac{1 - aT + qT^2}{(1-T)(1-qT)}$$

where a is an integer. We note that by using a Taylor expansion

$$\sum_{r=1}^{\infty} \frac{\#E(\mathbb{F}_{q^r})}{r} = \log\left(\frac{1 - aT + qT^2}{(1-T)(1-qT)}\right) = (q - a + 1)T + O(T^2)$$

we have

$$a = q + 1 - \#E(\mathbb{F}_q).$$

PROBLEM 10.1. Compute the congruent zeta function of an elliptic curve

$$\{[X : Y : Z]; YZ^2 = X(X-Z)(X+Z)\}$$

over $\mathbb{F}_p$ for a prime of your choice.

PROBLEM 10.2. Find a formula for a congruent zeta function of elliptic curves

$$\{[X : Y : Z]; YZ^2 = X(X-Z)(X-\lambda Z)\}$$

over $\mathbb{F}_q$ for $\lambda \in \mathbb{F}_q$.

PROBLEM 10.3. Compute the congruent zeta function of an elliptic surface of your choice.

PROBLEM 10.4. Compute the congruent zeta function of a homology plane of your choice. Compare the result with the congruent zeta function of a plane.

PROBLEM 10.5. Describe what happens to the conguent zeta function when we blow up an scheme.

PROBLEM 10.6. Let R be a commutative ring which is fninitely generated over $\mathbb{Z}$. Let $\varphi : R \to R$ be a ring homomorphism. Let us define "the semi direct product" $R \rtimes_\varphi \mathbb{N}$ as

$$R \times_\varphi \mathbb{N} = \langle R, \tau; \tau x = \varphi(x)\tau \qquad (\forall x \in R)\rangle$$

Compute the zeta function of a category (A-modules). Compare it with the category of the dynamical system $(\operatorname{Spec} R, \operatorname{Spec} \varphi)$.

PROBLEM 10.7. Is it possible to describe zeta functions of dynamical systems as zeta functions of categories?