COMMUTATIVE ALGEBRA

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07. Regular local ring is UFD(0) strategy.

Supplement:

Proposition 7.1. *Let A be a ring. Then* $\dim A = \sup_{\mathfrak{m}\in \mathrm{Spm}(A)} A_{\mathfrak{m}} = \sup_{\mathfrak{p}\in \mathrm{Spec}(A)} A_{\mathfrak{p}}$.

Definition 7.2. Let $\mathfrak{p}$ be a prime ideal of a ring A. Then we define its **height** $\mathrm{ht}\,\mathfrak{p}$ to e the supremum of the lengths of prime chaines

$$\mathfrak{p} = \mathfrak{p}_0 \supsetneq \mathfrak{p}_1 \supsetneq \mathfrak{p}_2 \supsetneq \cdots \supsetneq \mathfrak{p}_r.$$

We have $\mathrm{ht}\,\mathfrak{p} = \dim A_{\mathfrak{p}}$.

Definition 7.3. For an ideal I of a ring A, we define its height $\mathrm{ht}\, I$ to be

$$\mathrm{ht}\, I = \inf\{\mathrm{ht}\,\mathfrak{p} | I \subset \mathfrak{p} \in \mathrm{Spec}(A)\}$$

Definition 7.4. Let A be a ring and M be an A-module. Then a prime ideal $\mathfrak{p}$ of A is called an associated prime ideal of M if It is the annihilator ann(Mx of some $x \in M$. The associated primes of the A-module A/I are refereedd to as the prime divisorsof I.

Theorem 7.5. *Let A be a Noetherian ring, and $I = (a_1, \ldots, a_r)$ an ideal generated by r elements; then if $\mathfrak{p}$ is a minimal prime divisor of I we have* $\mathrm{ht}\,\mathfrak{p} \leq r$.

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Theorem 7.6. *Regular local ring $(A, \mathfrak{m})$ is UFD.*

Step 1.
Induction on $\dim(A)$.
If $\dim(A) = 0$, Then A is a field. Thus it is UFD.
Assume $\dim(A) > 0$. Take $x \in \mathfrak{m} \setminus \mathfrak{m}^2$. It suffices to prove:

(1) If $A[x^{-1}]$ is UFD, then A is UFD.
(2) $A[x^{-1}]$ is UFD.