COMMUTATIVE ALGEBRA

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04. Derivations and differentials

Lemma 4.1. *Let A be a commutative ring. Then for the polynomial ring $A = A[X_1, X_2, \dots, X_n]$ of n-variables over A, the module $\Omega^1_{B/A}$ of 1-differentials of B over A is equal to a free module generated by $dX_1, dX_2, \dots dX_n$. Namely, we have*

$$\Omega^1_{B/A} = AdX_1 \oplus AdX_2 \oplus \cdots \oplus AdX_n.$$

Lemma 4.2. *Let k be a ring. Let A, B be k-algebras. Then for any k-algebra homomorphism $\varphi : A \to B$ we have*

$$B \otimes_A \Omega^1_{A/k} \to \Omega^1_{B/k} \to \Omega^1_{B/A} \to 0$$

Lemma 4.3. *Let A be a commutative ring. Let B be a commutative A-algebra. Then for any ideal I of B, we have the following exact sequence:*

$$I/I^2 \to \Omega^1_{B/A}/I\Omega^1_{B/A} \to \Omega^1_{(B/I)/A} \to 0$$

where the first arrow maps $f \mod I^2$ to df.