# CONGRUENT ZETA FUNCTIONS. NO.7

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## 7.1. **Jacobi symbol.**

Definition 7.1. Let $m$ be a positive odd integer. Let us factor $m$:

$$m = \prod_i p_i^{e_i}$$

where $p_i$ are primes. Then for any $n \in \mathbb{Z}$, we define Jacobi symbols as follows

$$\left(\frac{n}{m}\right) = \prod_i \left(\frac{n}{p_i}\right)^{e_i}$$

We further define

$$\left(\frac{a}{p}\right) = 0 \text{ if } a \in p\mathbb{Z}.$$

Theorem 7.2 (quadratic reciprocity theorem). *For any positive odd integers $n, m$, we have*

$$\left(\frac{m}{n}\right)\left(\frac{n}{m}\right) = (-1)^{(m-1)(n-1)/4}.$$

Theorem 7.3. *Let $n$ be a postive odd integer. Then:*

(1) $\left(\frac{-1}{m}\right) = (-1)^{(m-1)/2}$.
(2) $\left(\frac{2}{m}\right) = (-1)^{(m^2-1)/8}$.

Exercise 7.1. $p = 113357$ is a prime. (You may use the fact without proving it.) Is there any integer $n$ such that

$$n^2 = 11351 \text{ in } \mathbb{Z}/p\mathbb{Z} \text{ ?}$$

If so, can you find such $n$?