

$$S(1\ 2) \cdot (S(12)\ S(1234)\ 1\ S((12)(34))\ S(123))$$

$$= (S(12)\ S(1234)\ 1\ S((12)(34))\ S(123)) \begin{pmatrix} 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 2 & 4 \\ 6 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 2 & 4 \\ 6 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 \end{pmatrix}$$

とおくと、 A の特性多項式は、

$$T^5 - 40T^3 + 144T (= T(T-6)(T-2)(T+2)(T+6))$$

$$A^2 = \begin{pmatrix} 20 & 16 & 0 & 0 & 0 \\ 16 & 20 & 0 & 0 & 0 \\ 0 & 0 & 6 & 6 & 24 \\ 0 & 0 & 2 & 10 & 24 \\ 0 & 0 & 3 & 9 & 24 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 0 & 20 & 52 & 144 \\ 0 & 0 & 16 & 56 & 144 \\ 120 & 96 & 0 & 0 & 0 \\ 104 & 112 & 0 & 0 & 0 \\ 108 & 108 & 0 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 656 & 640 & 0 & 0 & 0 \\ 640 & 656 & 0 & 0 & 0 \\ 0 & 0 & 120 & 312 & 864 \\ 0 & 0 & 104 & 328 & 864 \\ 0 & 0 & 108 & 324 & 864 \end{pmatrix}$$

$$S(12)^2 = 6 + 2S((12)(34)) + 3S(123)$$

$$S(12)^3 = 20S(12) + 16S(1234)$$

$$S(12)^4 = 120 + 104S((12)(34)) + 108S(123)$$

$$S(12)^5 = 656S(12) + 640S(1234)$$